Theories of Failure

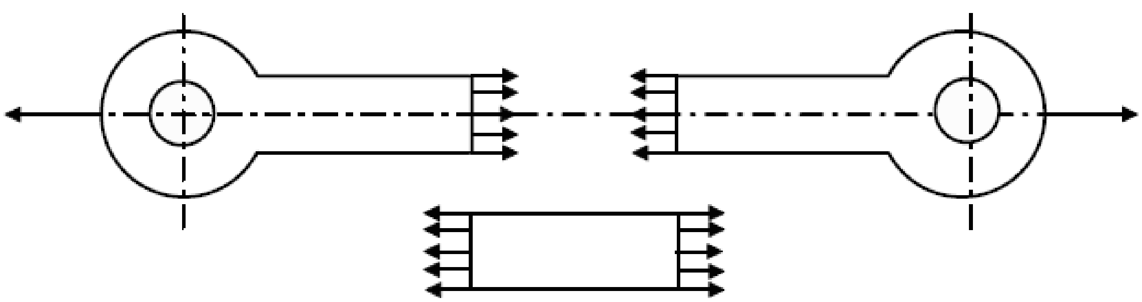
Instructional Objectives

At the end of this lesson, the students should be able to understand

- Types of loading on machine elements and allowable stresses.
- Concept of yielding and fracture.
- Different theories of failure.
- Construction of yield surfaces for failure theories.

Machine parts fail when the stresses induced by external forces exceed their strength. The external loads cause internal stresses in the elements and the component size depends on the stresses developed.

For example stresses developed in a link subjected to uniaxial loading is shown in figure,



Loading may be due to:

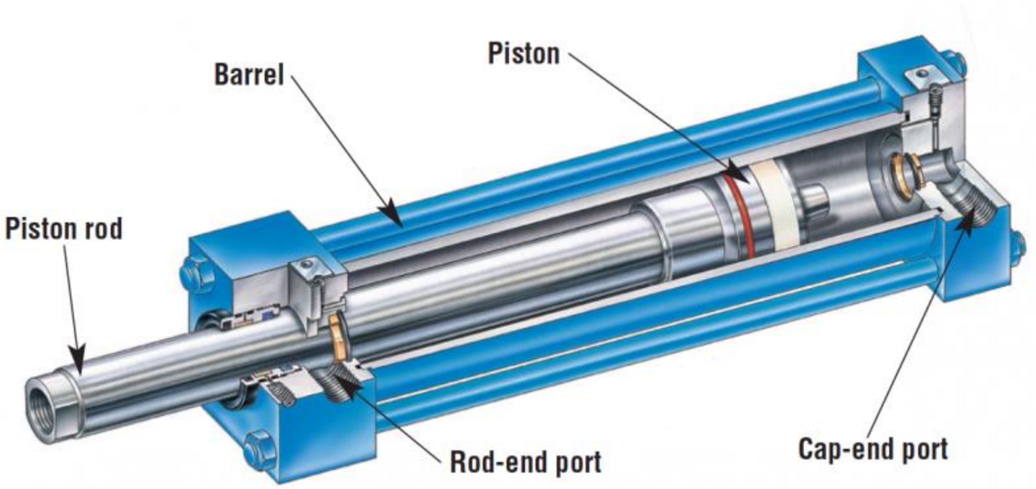
a) The energy transmitted by a machine element.
b) Dead weight.
c) Inertial forces.
d) Thermal loading.
e) Frictional forces.

In another way, load may be classified as:

a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.

b) Dynamic load-

- **Load may change in magnitude** for example, traffic of varying weight passing a bridge.
- **Load may change in direction**, for example, load on piston rod of a double acting cylinder.

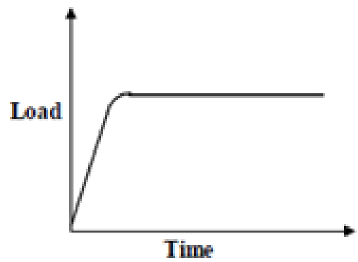


Double acting cylinder

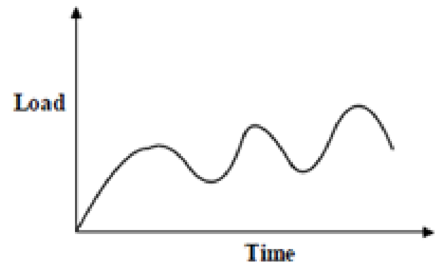
- **Vibration and shock** are types of dynamic loading.

The following figure shows load vs time characteristics for both static and dynamic loading of machine elements.

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University



Static Loading



Dynamic Loading

Allowable Stresses: Factor of Safety

Determination of stresses in structural or machine components would be meaningless unless they are compared with the **material strength**.

If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe.

The strength of various materials for engineering applications is determined in the laboratory with standard specimens.

For example, for tension and compression tests a round rod of specified dimension is used in a tensile test machine where load is applied until fracture occurs. This test is usually carried out in a **Universal testing machine**.

The load at which the specimen finally ruptures is known as **Ultimate load**
The ratio of load to **original** cross-sectional area is the **Ultimate stress**.

Similar tests are carried out for **bending**, **shear** and **torsion.**

The results for different materials are available in handbooks.

For design purpose an **allowable stress** is used in place of the critical stress to take into account the uncertainties including the following:

1) Uncertainty in loading.
2) Inhomogeneity of materials.
3) Various material behaviors. e.g. corrosion, creep.
4) Residual stresses due to different manufacturing process.
5) Fluctuating load (fatigue loading): Experimental results and plot-ultimate strength **depends on number of cycles**.

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

6) Safety and reliability.

For **ductile** materials (steel), the **yield strength** and for **brittle** (glass) materials the **ultimate strength** are taken as the critical stress.

An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e.

$$\frac{\text{Ultimate Stress}}{\text{Allowable Stress}} = \text{F.S.}$$

The ratio must always be greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

Theories of failure

When a machine element is subjected to a system of complex stress system, it is important to predict the mode of failure so that the design methodology may be based on a particular **failure criterion**.

Theories of failure are essentially a set of failure criteria developed for the ease of design.

In machine design an element is said to have failed if it ceases to perform its function.

There are basically two **types of mechanical failure**:

(a) **Yielding**- This is due to excessive inelastic deformation rendering the machine part unsuitable to perform its function. This mostly occurs in **ductile** materials.

(b) **Fracture**- in this case the component tears apart in two or more parts. This mostly occurs in **brittle** materials. There is no sharp line of separation between ductile and brittle materials.

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

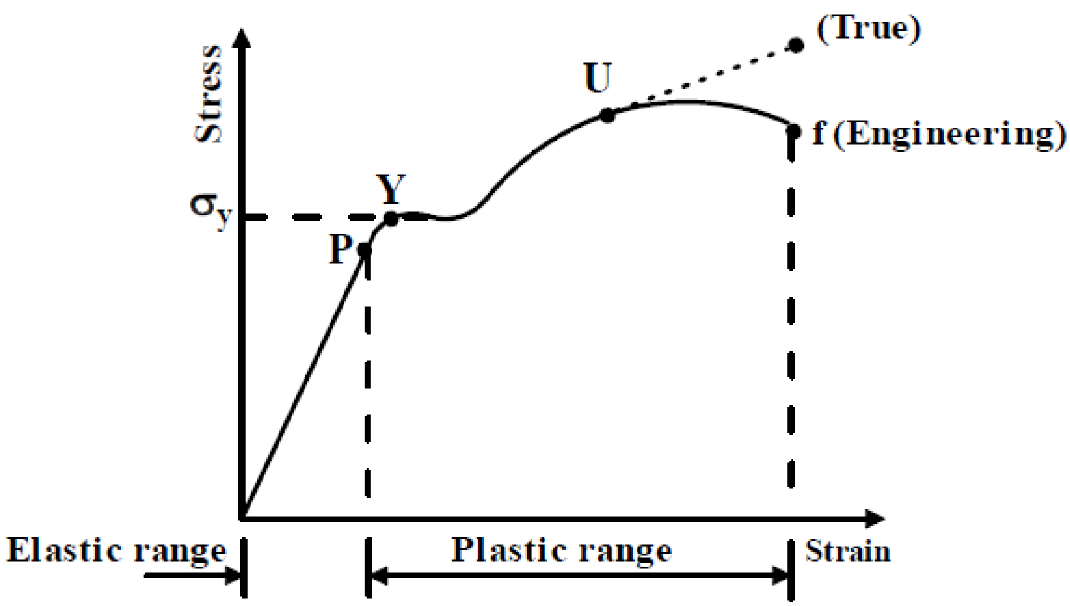
However a rough guideline is that if percentage elongation is **less than 5%** then the material may be treated as **brittle** and if it is **more than 15%** then the material is **ductile**.

However, there are many examples when a **ductile** material may fail by **fracture**. This may occur if a material is subjected to

(a) Cyclic loading.
(b) Long term static loading at high temperature.
(c) Impact loading.
(d) Work hardening.
(e) Severe quenching.

Yielding and fracture can be visualized in a typical tensile test as shown in the clipping

Typical engineering stress-strain relationship from simple tension tests for same engineering materials are shown



Stress-strain diagram for a ductile material e.g. low carbon steel

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

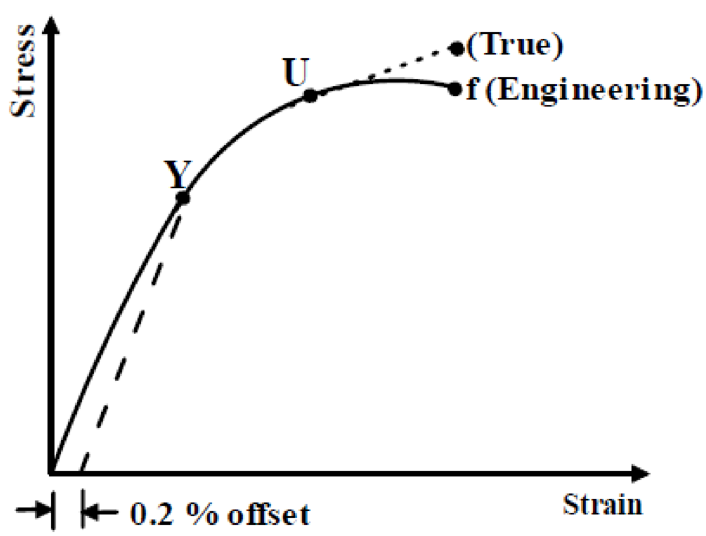
- For a typical ductile material as shown in figure there is a definite yield point where **material begins to yield more rapidly without any change in stress level**. Corresponding stress is $\boldsymbol{\sigma_y}$.
- Close to yield point is the proportional limit which marks the transition from elastic to plastic range.
- Beyond elastic limit yielding would continue without further rise in stress.

For most ductile materials, such as, low-carbon steel beyond yield point the stress in the specimens rises upto a peak value known as **ultimate tensile stress $\boldsymbol{\sigma_u}$** .

- Beyond **ultimate tensile stress $\boldsymbol{\sigma_u}$** the specimen starts to neck-down

  i.e. the reduction in cross-sectional area.

However, the stress-strain curve falls till a point where fracture occurs.



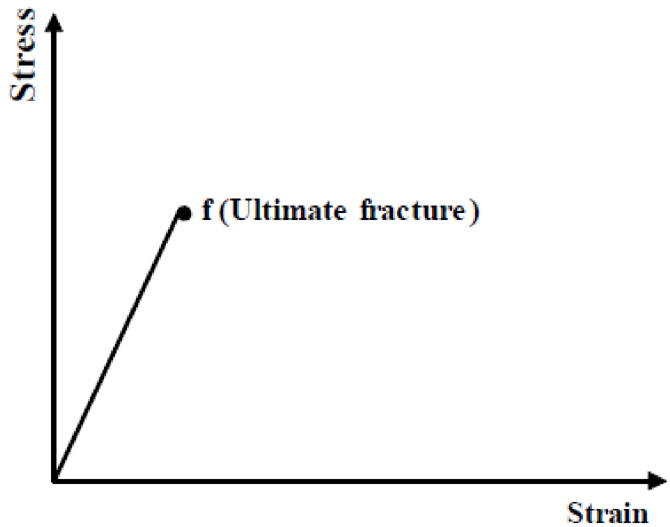
Stress-strain diagram for low ductility.

The drop in stress is obvious since original cross-sectional area is used to calculate the stress.

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

If instantaneous cross-sectional area is used the curve would rise as shown in figure (true).

For a material with low ductility there is no definite yield point and usually off-set yield points are defined for ease.



Stress-strain diagram for a brittle material

For a brittle material stress increases linearly with strain till fracture occur.

Yield criteria

There are many yield criteria, going as far back as Coulomb (1773).

- Maximum principal stress theory (Rankine theory)
- Maximum principal strain theory (St. Venant's theory)
- Maximum shear stress theory (Tresca theory)
- Maximum strain energy theory (Beltrami's theory)
- Distortion energy theory(von Mises yield criterion)

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

One of them will be discussed briefly here.

Maximum principal stress theory (Rankine theory)

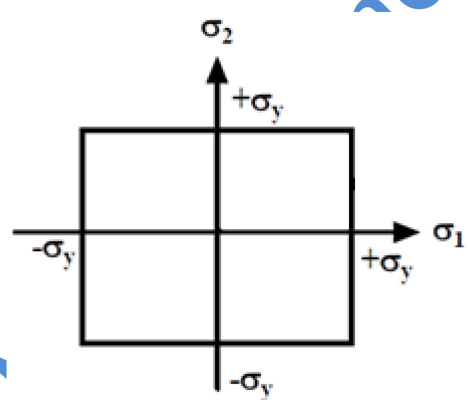
According to this, if one of the principal stresses σ1 (maximum principal stress) , σ2 (minimum principal stress) exceeds the yield stress, yielding would occur.

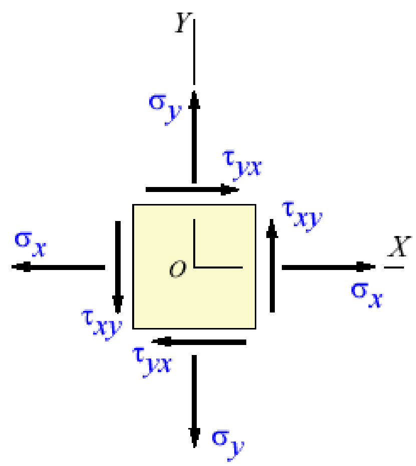
In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$$\sigma_1 = \pm\, \sigma_y$$

$$\sigma_2 = \pm \sigma_y$$

Using this, a yield surface may be drawn, as





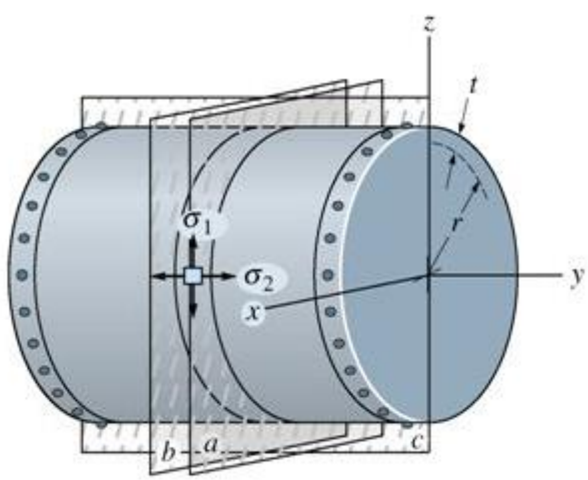
Stresses in given coordinate system

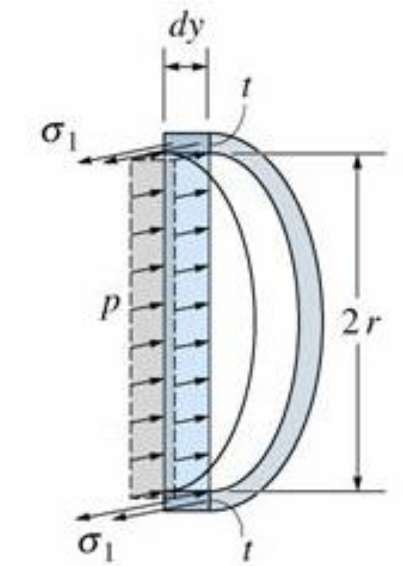
Yielding occurs when the state of stress is at the boundary of the rectangle

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

For example, the state of stress of a thin walled pressure vessel.

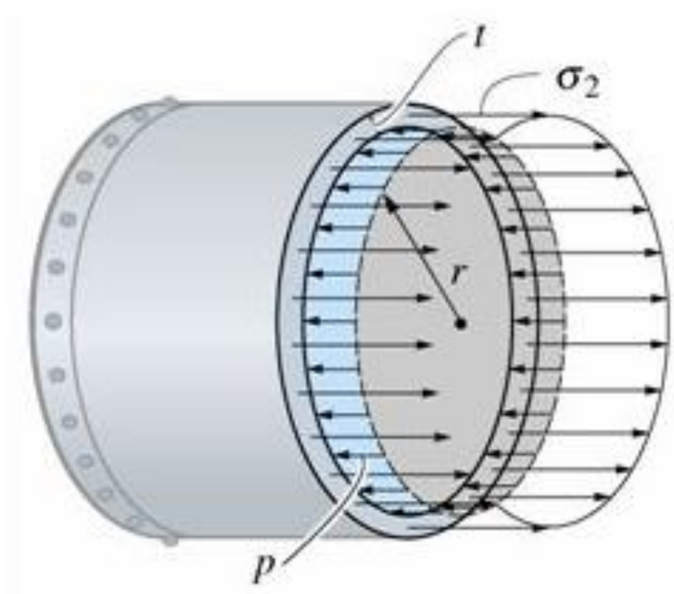
- $\sigma 1$ being the circumferential or hoop stress
- $\sigma 2$ the axial stress





$$\sigma_1(2t\,dy) = p(2r\,dy)$$

$$\sigma_1 = \frac{pr}{t} \quad \textbf{(Hoop Stress)}$$



$$\sigma_2(2\pi rt) = p(\pi r^2)$$

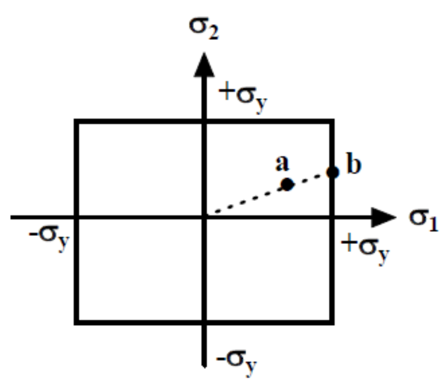
Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

$$\sigma_2 = \frac{pr}{2t} \quad \textbf{(Longitudinal Stress)}$$

Here

$$\sigma_1 = 2\sigma_2,$$

As the pressure in the vessel increases the stress follows the dotted line. At a point (say) **a**, the stresses are still within the elastic limit but at **b**, σ_1 reaches σ_y although σ_2 is still less than σ_y.

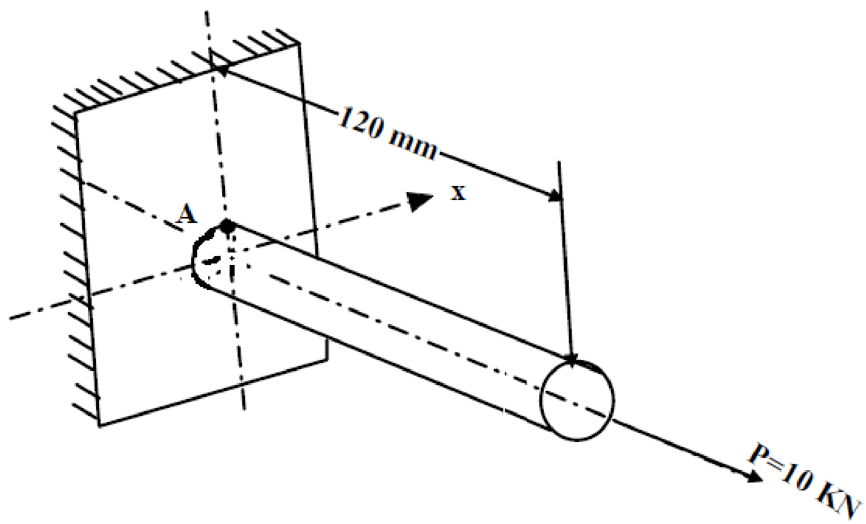


*Yielding will then **begin** at point b*

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

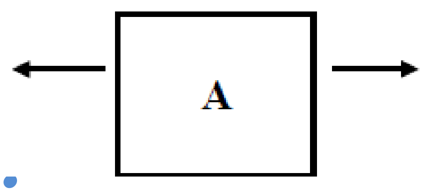
Exercise problem

A cantilever rod is loaded as shown in the figure. If the tensile yield strength of the material is 300 MPa **determine** the rod diameter using Maximum principal stress theory.



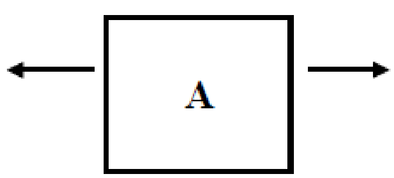
At the beginning it is necessary to identify the mostly stressed element.

Axial normal stress is the same throughout the length of the rod.



$$P\Big/\left(\frac{\pi d^2}{4}\right) \quad \textbf{(Axial stress)}$$

Substituting values



$$\left(\frac{12732}{d^2}\right)$$

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University

$$\sigma_1 = \frac{12732}{d^2}$$

$$\sigma_2 = 0$$

Setting $\sigma_1 = \sigma_Y$ we get **d = 6.5 mm**.

Dr. Mohamed Ahmed Aziz
Mechanical Eng. Dep. Faculty of Engineering, Suez University